particular technique, it would be advisable to verify its reliability by an additional measuring method.

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A Simple Correlation for Incipient Turbulent Boundary-Layer Separation due to a Skewed Shock Wave

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Introduction

A FREQUENTLY encountered form of three-dimensional shock wave-turbulent boundary-layer interaction is that due to a wedge mounted normal to a surface in a supersonic stream. Such a configuration is representative of a sharp-edged fin on a flying vehicle, an axial compression corner in an air breathing engine inlet or a wind-tunnel diffuser, and end wall conditions for a row of blades in turbomachinery.

The configuration is illustrated in Fig. 1. Θ is the wedge half angle or flow deflection angle and β is the wedge shock wave angle. The shock wave is skewed to the turbulent boundary layer on the flat plate on which the wedge is mounted. Of particular interest for practical applications is the condition of incipient separation of the turbulent boundary layer, defined by the deflection angle Θ_i.

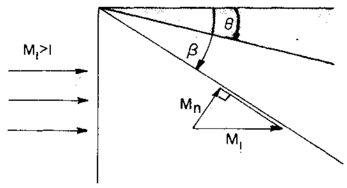

Fig. 1 Skewed shock wave on flat plate.

Theory

McCabe[1] develops a simple approximate theory for conditions across the skewed shock wave, which relates the deflection of the flow at the surface to the flow at the edge of the turbulent boundary layer, based on the assumption that all the vorticity in the boundary layer upstream of the shock wave is convected with the freestream velocity. He obtains

$$\tan \varepsilon = \frac{\cos^2(\beta-\Theta)-\cos^2\beta}{\cos^2\beta \tan(\beta-\Theta)} \tag{1}$$

where ε is the angle between the surface flow and the edge flow.

McCabe makes the reasonable assumption that incipient separation occurs when the surface flow becomes aligned with the shock wave, i.e. when $\varepsilon = \beta - \Theta_i$, for which condition Eq. (1) becomes

$$\cos\beta = \cos^2(\beta-\Theta_i) \tag{2}$$

With the assumption of small angles corresponding to $M_1 \gg 1$, Eq. (2) can be approximated to a first order by

$$\Theta_i \simeq (1-2^{1/2}/2)\beta = 0.293\beta \tag{3}$$

For practical purposes it is useful to relate the incipient deflection angle Θ_i to the freestream Mach number M_1. From oblique shock wave theory, the relationship between deflection angle, shock wave angle and Mach number is

$$\cot\Theta = \tan\beta\left[\frac{[(\gamma+1)/2]M_1^{\,2}}{M_1^{\,2}\sin^2\beta-1}-1\right] \tag{4}$$

which, for small angles and $\gamma = 1.4$, approximates to

$$M_1^{\,2}\beta^2-1 = 1.2M_1^{\,2}\Phi\beta-(M_1^{\,2}\beta^2-1)\Theta\beta \tag{5}$$

Substituting Eq. (3) for the incipient condition into Eq. (5) results in

$$M_1\Theta_i = 0.364[1+0(\Theta_i^{\,2})] \simeq 0.364 \tag{6}$$

Thus, from McCabe's theory for $M_1 \gg 1$ the incipient separation angle Θ_i is inversely proportional to Mach number. Now, the component of Mach number normal to the oblique shock wave is $M_n = M_1 \sin\beta \simeq M_1\beta$. For the incipient condition, by substitution of Eqs. (3) and (6) one obtains

$$M_n = 3.42M_1\Theta_i = 1.24$$

for which the pressure rise is $P_i/P_1 = 1.63$, a constant value independent of Mach number.

Comparison with Experiment

Incipient separation data by McCabe[1] and Lowrie[2] are given in Table 1 along with the corresponding shock-wave angles, normal Mach number components, and pressure rises.

The data, although over a limited Mach number range, bear out the near constancy of the pressure rise and of $M_1\Theta_i$ although the values are somewhat lower than that of Eq. (6). Better agreement with experiment is given by

$$M_1\Theta_i = 0.30 \tag{7}$$

This correlation along with the experimental values from Table 1 are shown in Fig. 2 in which the bars across the

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Table 1 Incipient separation data

M_1	$\Theta_i^{(o)}$	$\beta^{(o)}$	M_n	P_i/P_1	$M_1\Theta_i$
1.96[a]	8.5	38.5	1.22	1.58	0.290
2.50[b]	7.5	29.5	1.23	1.60	0.325
2.94[a]	5.5	24.0	1.20	1.50	0.282
3.44[b]	5.0	20.5	1.20	1.50	0.300

[a] Data from Ref. 1.
[b] Data from Ref. 2.

experimental points represent a conservative assumption of $\pm 0.5°$ accuracy of the data.

Substituting $\sin\beta = M_n/M_1$ for the shock angle into the oblique shock wave relation, and making the small angle approximation corresponding to $M_1 \gg M_n$ or 1, Eq. (4) is expressed, to a first order, by

$$M_1\Theta = [M_n^2-1]\bigg/\left[\left(\frac{\gamma+1}{2}\right)M_n\right] \tag{8}$$

For the incipient case of Eq. (7), one obtains $M_n = 1.20$ corresponding to which $P_i/P_1 = 1.50$. The error in M_n for the incipient case as calculated from Eq. (8) compared with the exact value given by Eq. (4) is less than 3% for $M \geqq 2$.

It is of interest to note that incipient turbulent boundary-layer separation for normal shock interaction occurs at a Mach number of about 1.3 according to Fage and Sargent,[3] for which the pressure rise is about 1.80. This suggests that a skewed shock gives rise to somewhat earlier separation than a normal shock. A conjecture for this behavior is that the surface streamlines are forced into a complete reversal in direction in the two-dimensional case, whereas with a skewed shock they are only pressed into a small change in direction in the process of coalescing along a separation line.

Summary and Discussion

In summary, a simple correlation is obtained for incipient turbulent boundary-layer separation due to the impingment of a skewed shock wave, as follows:

$$M_1\Theta_i = 0.30 \qquad P_i/P_1 = 1.50$$

Thus incipient separation is associated with an approximately constant pressure rise independent of Mach number and the incipient separation deflection angle varies inversely with Mach number at least over the range of available experimental data. The validity of this correlation needs checking by experimental data at Mach numbers above 3.5.

It is interesting to note that this behavior is contrary to that for two-dimensional turbulent boundary-layer separation for which the incipient deflection angle increases with increasing Mach number, and the corresponding pressure rise increases at an even steeper rate.

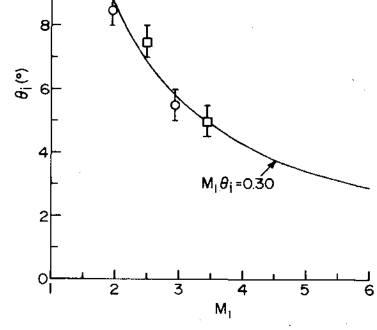

Fig. 2 Correlation for flow deflection angle for incipient separation.

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Near-Field Trajectory of Turbulent Jets Discharged at Various Inclinations into a Uniform Crossflow

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Introduction

THE deflected flow of turbulent jets in cross streams has been extensively investigated by environmental engineers in connection with the discharge of effluents from chimney stacks into the atmosphere or of liquid pollutants into natural bodies of water. Their interest inclines toward the study of dispersion of effluents or pollutants at far field, away from the discharge point. Recent interest in such interacting flows also has arisen from the design of V/STOL aircrafts. During the transition from hovering to forward flight, such aircraft produce an analogous deflected-jet flow, which, in turn, can have serious consequences on the dynamic behavior of the aircraft; the interest here is the near-field flow of deflected jets.

A systematic experiment was conducted by Platten and Keffer[1] of axisymmetric turbulent jets discharging at various angles of inclination into a uniform cross stream having various ratios between jet and stream velocities. On the basis of their results, it is shown herein that jet trajectories in the near field follow a universal law by which the variations in both the angle of inclination and the velocity ratio can be accommodated.

Turbulent Jets Ejected Normally into Cross Streams

The flow of a turbulent jet discharging normally into a cross stream was considered by Pratte and Baines[3] to exhibit three sequential patterns of development: 1) a potential core which exists before the turbulent shear region developed along the jet boundary reaches the centerline of the jet; 2) a zone of maximum deflection in the near field where rapid entrainment, similar to that for a free jet, occurs to cause the jet to bend over; 3) a vortex zone in the far field where the flow is governed mainly by the motion of a turbulent vortex pair formed following the bending over.

As discussed by Pratte and Baines,[3] the centerline jet trajectory, especially in the near field, can be obtained in simple form from a dimensional argument on the basis of flow similarity

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Index category: Jets, Wakes, and Viscid-Inviscid Flow Interactions.

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